A NOTE ON CURVILINEARITY, DEMOCRACY, AND VICTORY

Alexander B. Downes
Department of Political Science
Duke University
October 1, 2009

In their recent letter to the editor of *International Security*, Dan Reiter and Allan Stam seek to rebut some of the criticisms I made of their work on democracy and victory in an article that appeared in the Spring 2009 issue of the journal.[1] One of their rebuttals concerns the question of whether the relationship between regime type and war outcomes for war initiators is curvilinear. This note provides more detail on this issue than was possible in my published response to their letter.[2]

REITER AND STAM'S ANALYSIS IN *DEMOCRACIES AT WAR*

Reiter and Stam divide political regimes into three categories—democracies, mixed regimes (moderately repressive non-democracies), and autocracies (highly repressive non-democracies)—and argue that democratic initiators are most likely to win, followed by autocracies, with mixed regimes being the least likely to achieve victory.[3] Reiter and Stam

---

[1] Dan Reiter and Allan C. Stam, "Correspondence: Another Skirmish in the Battle over Democracies and War," *International Security*, Vol. 34, No. 2 (Fall 2009). For my article, see Alexander B. Downes, "How Smart and Tough Are Democracies? Reassessing Theories of Democratic Victory in War," *International Security*, Vol. 33, No. 4 (Spring 2009), pp. 9-51. The focus of much of my critique was Dan Reiter and Allan C. Stam, *Democracies at War* (Princeton, N.J.: Princeton University Press, 2002).

[2] My response also appears in *International Security*, Vol. 34, No. 2 (Fall 2009).

[3] Democracies are considered to be states that receive a score of 7 or above on the Polity index, which ranges from -10 to +10. Mixed regimes score between -6 and +6, and dictatorships score -7 or below. The examples of autocratic military ineffectiveness that Reiter and Stam give throughout *Democracies at War*, however, convey the impression that autocracies—such as Saddam Hussein's Iraq, Stalinist Russia, and Hafez al-Assad's Syria—rather than mixed regimes are the least capable type of authoritarian state. Reiter and Stam, *Democracies at War*, pp. 24, 70. On the military disadvantages of personalist dictatorships, see, for example, Risa Brooks, *Political-Military Relations and the Stability of Arab Regimes*, Adelphi Paper 324 (Oxford: Oxford University Press, 1998); and James T. Quinlivan, "Coup-proofing: Its Practice and Consequences in the Middle East," *International Security*, Vol. 24, No. 2 (Fall 1999), pp. 131-165. For a similar and quite sophisticated argument regarding how institutional differences among non-democracies affect their conflict behavior, including wartime performance, see Jessica L. Weeks, "Leaders, Accountability, and Foreign Policy in Non-Democracies," Ph.D. dissertation, Stanford University, 2008.

criticize my test of democratic efficacy for neglecting this curvilinear relationship and instead assuming a linear relationship between democracy and victory, claiming that "it is not surprising that a linear model applied to a curvilinear relationship failed to find statistically significant results."[4] On the face of it this is an odd claim because Reiter and Stam performed precisely the same type of analysis—utilizing a linear specification of democracy and a probit model—in *Democracies at War*.[5] Indeed, I followed Reiter and Stam's procedure in my article and used an interaction term between democracy and initiation to gauge whether democracies were significantly more likely than non-democracies to win the wars they start. Because I added draws to the dataset, I used an ordered probit model (designed for dependent variables with more than two ordered categories) rather than the probit model used by Reiter and Stam. If forcing an underlying curvilinear relationship to take a linear form is responsible for producing the insignificant results for democratic war initiators in my analysis, then Reiter and Stam's linear treatment of democracy should also yield insignificant results. Their results, however, are significant, which suggests that neglecting to model an underlying curvilinear relationship between regime type and victory is not the cause of statistical insignificance of democracy in my analysis.

In addition to this linear specification, however, Reiter and Stam also tested for a curvilinear relationship between democracy and victory for war initiators. To test for such a relationship, in *Democracies at War* Reiter and Stam used fractional polynomials (FPs), which are designed to detect non-linear effects of continuous independent variables. Many analysts simply insert squared or cubed values of the independent variable of interest to test for

---

[4] Reiter and Stam are not actually concerned with the type of model or estimator I use, but rather the functional form I assume for democracy. Any model can estimate non-linear effects given the proper transformation of the independent variable.

[5] Reiter and Stam, *Democracies at War*, p. 45, models 1 and 4.

curvilinearity; these terms do not always fit well, however, especially at extreme values of the independent variable.[6] Moreover, Reiter and Stam had a particular curve shape in mind: one that starts at a moderately high probability of victory (autocracies), then decreases as democracy begins to increase (mixed regimes), but finally bends upward again as one reaches the highest values for democracy. Fractional polynomials offer greater flexibility and fit to the data because in addition to squared and cubed terms, they use fractional powers as well.[7] The model then reports the coefficients for the FPs that best fit the data, as well as a measure of whether the curvilinear model is a significant improvement over a linear one.

In *Democracies at War*, Reiter and Stam selected an FP model with two terms: x-½ and x-½$(\ln(x))$, where x consisted of a transformation of states' democracy scores equal to (Polity × Initiator + 11)/10.[8] Some form of transformation is needed because FP models work only for values of independent variables greater than zero; the combined Polity index, however, ranges from -10 to +10 (hence the reason for adding eleven to the Polity score). Reiter and Stam's procedure is equivalent to using the *fracpoly* command in Stata with the variable Polity × Initiator (plus controls) and specifying that the model use the powers (-0.5,-0.5). The *fracpoly* model automatically adds eleven (to make Polity × Initiator strictly positive) and divides by ten (to reduce the variance). It then produces two terms, identical to Reiter and Stam's: x-½ and x-½$(\ln(x))$. Model 1 in table 1 below shows the results of the *fracpoly* regression, which are identical to those in *Democracies at War*.[9] Each of the FP terms is statistically significant, which

---

[6] Patrick Royston and Douglas G. Altman, "Regression using Fractional Polynomials of Continuous Covariates: Parsimonious Parametric Modeling," *Applied Statistics*, Vol. 43, No. 3 (1994), pp. 429-430.

[7] For example, by default the *fracpoly* model in Stata 10 uses the following set of powers: {-2, -1, -0.5, 0, 0.5, 1, 2, 3}. Patrick Royston and Douglas G. Altman, "Using Fractional Polynomials to Model Curved Regression Relationships," *Stata Technical Bulletin*, Vol. 21 (September 1994), p. 13.

[8] Reiter and Stam, *Democracies at War*, p. 41.

[9] See Reiter and Stam, *Democracies at War*, p. 45, table 2.2, model 5. See also Reiter and Stam, "Democracy, War Initiation, and Victory," *American Political Science Review*, Vol. 92, No. 1 (June 1998), p. 385, table 2, model 5.

Reiter and Stam claim supports the hypothesis that a curvilinear relationship exists between regime type and victory for war initiators.

[Table 1 about here]

There are three problems that undermine the validity of these results. The first concerns the formula for x in the paragraph above. Although the FP exercise is meant to test whether the relationship between regime type and victory for war *initiators* is curvilinear, using this formula assigns states that are targets a value of 1.1, the same as the score for an initiator with a Polity score of zero: (Polity $\times$ 0 + 11)/10 = 1.1. The FP terms—rather than excluding targets, as they should—include targets but place them all exactly in the middle of the Polity spectrum. Indeed, visual inspection of belligerents' Polity scores plotted against the partial effect of Polity on the probability of victory (see figure 1) shows a large cluster of observations at zero when only a single state in the dataset has a score of zero for regime type. Thus, although the FPs are statistically significant, because they are constructed improperly to include war targets these results cannot be used as evidence to support the hypothesis of a curvilinear effect of democracy on victory for initiators.

[Figure 1 about here]

The second, more important problem with Reiter and Stam's FP analysis in *Democracies at War* is that statistical significance of the FP variables is not the proper metric by which to judge whether a curvilinear relationship exists between regime type and war outcomes. The correct way to make this determination is to compare the model fit of the curvilinear specification to that of the linear specification and check whether the former is a significant

4

improvement over the latter.[10] The *compare* command used in conjunction with *fracpoly* in *Stata* produces a matrix that compares the deviance (-2 × log-likelihood) of the best-performing FP model to the deviance of a model containing the linear specification of the variable in question, and the degree to which the difference in deviance is statistically significant. If no FP model represents a significant improvement in fit over the linear specification, then the linear version is preferred.

In the case of Reiter and Stam's analysis of curvilinearity in the relationship between regime type and victory in *Democracies at War*, simple observation is enough to show that there is no evidence of curvilinearity. Comparing the FP model (p. 45, model 4) with a model in which the effect of democracy on war outcomes is treated as linear (p. 45, model 5) shows that the log-likelihoods for each are identical (-64.9), meaning that the deviance statistic is also identical (129.8) and thus there is no difference in the two models' explanatory power. Again, although it is true that the two FPs are statistically significant, this does not mean that treating democracy as curvilinear fits the data better than treating it as linear. In fact, the FP results indicate that FPs are not needed. Thus, the statistical evidence in *Democracies at War* does not support the existence of a curvilinear relationship between regime type and victory.

A third, more minor problem with Reiter and Stam's FP analysis in *Democracies at War* is that the powers they chose for their FPs are *not* the ones that best fit the data. Many combinations of fractional terms form curves shaped like the one Reiter and Stam desired, and they do not explain why they selected this one in particular. On closer inspection, it turns out that a model with powers (-0.5,-0.5) is not the optimal combination. I re-ran Reiter and Stam's model, but rather than specifying the powers for the FPs, I allowed *fracpoly* to select the most

---

[10] Patrick Royston and Willi Sauerbrei, *Multivariable Model-Building: A Pragmatic Approach to Regression Analysis Based on Fractional Polynomials for Modelling Continuous Covariates* (Chichester, U.K.: John Wiley, 2008), pp. 82-83.

efficient powers (see model 2 in table 1). The model chose terms of different degree (0,1) than

Reiter and Stam did, with a total deviance of 129.19.[11] Although this represents a small

improvement over the deviance of Reiter and Stam's preferred model (129.73), the key point is

that neither version improves significantly on the deviance of a model that treats Polity as linear

(129.77).[12] The difference in deviance between the best-fitting FP model (with degrees 0,1) and

the version with Polity as linear is 0.58, with a *p*-value of 0.90. Thus, although there is an FP

model that fits the data better than Reiter and Stam's it still does not provide evidence of

curvilinearity in the relationship between democracy and victory.

Why did Reiter and Stam conclude that the curvilinear transformation of Polity

significantly improved the fit of their model over a version that treated Polity as linear?[13] The

main reason appears to be that Reiter and Stam used the wrong formula to calculate the deviance

statistic. Examination of the formula they used indicates that Reiter and Stam compared the fit of

the FP model not to that of a probit model with Polity entered as a linear predictor, but rather to a

linear *model* (i.e., ordinary least squares). The formula they cite for calculating the deviance is

n(1 + log(2πRSS/n)), "where n is the sample size and RSS is the residual sum of squares

estimated by fitting a linear model."[14] The source they cite as a reference on FP models,

however, clearly states: "We assume that all models are to be fitted by maximum likelihood." It

further defines deviance as "$D = -2 \times$ log-likelihood," and defines the difference in deviance

(also referred to as *gain*) between linear and non-linear models as the deviance of the model with

the covariate treated as linear minus the deviance of the model with the covariate treated as non-

---

[11] As noted above, the fit of FP models is evaluated by comparing the reduction in deviance (from a perfectly fitting model) achieved by the use of various FP terms. The formula for deviance is simply $-2 \times$ log-likelihood.
[12] One of the FPs in the better-fitting model is also insignificant.
[13] Reiter and Stam, *Democracies at War*, pp. 46, 215 note 84.
[14] See Reiter and Stam, *Democracies at War*, p. 215 note 84; and "Democracy, War Initiation, and Victory," pp. 384-85 note 13.

linear.[15] It thus seems likely that Reiter and Stam misinterpreted the directions for how to test whether a curvilinear specification of democracy represents a statistically significant improvement over a linear specification of democracy, which could explain why they concluded that there is a significant difference between the two.

Given that they used the incorrect formula to calculate deviance, one other factor may have led Reiter and Stam to conclude that their FP model was superior to treating the relationship between democracy and victory as linear. Reiter and Stam write that suppressing the constant term in the FP model increases the significance of the fractional terms, and that they used this improved model to calculate the deviance from the linear model.[16] The reason they give for suppressing the constant is that it is not significant; why this is a legitimate reason to omit the constant they do not explain. Because estimating a model without a constant assumes that the y-intercept is zero, the only reason for omitting the constant is if the analyst is very confident that the value of the dependent variable is zero if all of the independent variables are zero.[17] This is rarely the case, however, and if the intercept is not zero, suppressing the constant results in biased coefficient estimates. Thus econometrics texts counsel extreme caution about omitting the constant.[18] In the case of war outcomes, for example, the predicted value of victory if all the independent variables are zero is 0.21, which undercuts the case for dropping the constant.[19] Moreover, suppressing the constant in Reiter and Stam's FP model does not lower the deviance

---

[15] Royston and Altman, "Regression using Fractional Polynomials of Continuous Covariates," p. 435. Gain is distributed Chi²-distributed with m degrees of freedom, m being equal to the number of powers in the FP model.

[16] Reiter and Stam, "Democracy, War Initiation, and Victory," p. 385 note 14.

[17] In other words, the regression line intersects the origin.

[18] For example, Gujarati writes "one needs to exercise great caution in using the zero intercept regression model. *Unless there is very strong a priori expectation*, one would be well advised to stick to the conventional, intercept-present model." Damodar N. Gujarati, *Basic Econometrics*, 3rd ed. (New York: McGraw-Hill, 1995), 159 (italics in original). Gujarati notes that dropping the constant may yield more precise estimates of the coefficients "if the intercept is in fact absent," but only at the risk of biased coefficients and specification error if it is not.

[19] I obtained this estimate by running a model with Reiter and Stam's FPs and using CLARIFY to set all the independent variables to zero and obtaining the predicted probability of victory. Because CLARIFY uses simulations, multiple repetitions of this exercise produced slightly different probabilities, all in the vicinity of 0.2.

compared to the linear specification. As already noted, the deviance of the model with democracy treated linearly is 129.8; the constant in this model is highly significant and thus Reiter and Stam's argument for suppressing it does not apply. Suppressing the constant in the FP model produces a deviance of 130.3. Not only does omitting the constant seem unwarranted on methodological grounds, it also fails to produce a curvilinear model with better explanatory power than a linear model.

REITER AND STAM'S ANALYSIS IN THE CORRESPONDENCE

As I pointed out in our correspondence, Reiter and Stam use a different set of FPs for the analysis they present in table 2 of their letter than they did in their book, having now correctly constructed them so as to exclude war targets.[20] This correction deals with the first critique I made above. However, the second critique still stands: Reiter and Stam continue to assert (incorrectly) that statistical significance of the FPs is sufficient to establish that there is a curvilinear relationship between democracy and victory. All sources on FP modeling that I have consulted are clear that this is false: evidence of curvilinearity exists only if the FP model explains more of the variance—judged by whether the difference in model deviances is statistically significant—than a model with the same covariate treated as linear.[21] Moreover, as I demonstrated in my contribution to the correspondence, Reiter and Stam's FP model explains *less* of the variance than the linear specification when draws are included or excluded even using

---

[20] The formulas are now x-½ × initiator and x-½(ln(x)) × initiator, with x consisting of Polity + 11.
[21] Royston and Altman, "Regression using Fractional Polynomials of Continuous Covariates"; Royston and Altman, "Using Fractional Polynomials to Model Curved Regression Relationships"; Patrick Royston and Willi Sauerbrei, "A New Approach to Modelling Interactions between Treatment and Continuous Covariates in Clinical Trials by Using Fractional Polynomials," *Statistics in Medicine*, Vol. 23 (2004): 2509-2525; and Royston and Sauerbrei, *Multivariable Model-Building*.

the corrected FPs. I conclude, therefore, that there was not, and still is not, evidence of a

significant curvilinear relationship between democracy and war outcomes for war initiators.[22]

OTHER METHODS OF CHECKING FOR CURVILINEARITY

There are other ways to check for curvilinearity, however. Model 3 in table 1, for example,

shows the results when a squared term for Polity × Initiator is added to Reiter and Stam's

baseline probit model.[23] As is evident, neither the linear nor the squared term is significant, and

the improvement in deviance over the linear treatment (0.48) is minimal.

Another way to check for curvilinearity is to split the sample and include only those

states that initiated wars. This strategy obviates the need for interaction terms and allows the

analyst to use *fracpoly* with Polity as the variable of interest (plus controls). Models 4 and 5 in

table 1 compare the results obtained using the linear specification of democracy versus the

curvilinear one. Given the reduced sample size ($N = 86$), the strategy dummy variables cause

problems, leading one or more to be dropped from the estimation owing to collinearity and

causing many observations to be completely determined. For this reason I omit them from the

analysis.[24] Once again, these models provide no evidence of curvilinearity: only one of the FPs is

significant, and the deviance for the FP model is larger than that of the linear model.[25] It is

---

[22] It is not feasible to assess the validity of my third critique—that the FPs Reiter and Stam used were not the best fitting ones—with the new FPs. The reason has to do with the order of multiplication in the new formula: the corrected FPs now multiply by initiator at the end of the equation (after the mathematical transformations) rather than the beginning. Inserting Polity × Initiator into the fracpoly model and specifying powers -0.5,-0.5 would produce the old FPs: (Polity × Initiator + 11/10)-½, and (Polity × Initiator + 11/10)-½ × ln(Polity × Initiator + 11/10). It is likely that Reiter and Stam transform the variables separately and then insert them into a regular probit model. This procedure makes it exceedingly cumbersome to compare the explanatory power of different combinations of powers because one would have to transform them each by hand and enter them into successive probit models and compare the results. There are 44 possible combinations.

[23] To facilitate this test, I use a version of Polity than varies from 1 to 21 rather than -10 to +10.

[24] This choice does not affect the results; the difference between the two models is insignificant whether or not these variables are included.

[25] Inserting Polity and Polity squared does not produce significant results either.

difficult to avoid the conclusion that the evidence simply does not support a curvilinear

relationship between regime type and victory for war initiators.

	1 Fracpoly probit model, politics × initiator, powers set to -0.5, -0.5	2 Fracpoly probit model, politics × initiator, powers selected by fracpoly	3 Probit model, squared term inserted	4 Probit model, initiators only, strategy variables dropped	5 Fracpoly probit model, initiators only, strategy variables dropped
Table 1 Testing for Curvilinearity in the Relationship Between Regime Type and War Outcomes (DV = Win/Lose; Reiter and Stam's original codings and cases)					
Fractional Term 1	-3.67# (1.88)	-0.48 (0.45)	-	-	0.02 (0.02)
Fractional Term 2	-1.07# (0.57)	1.33# (0.73)	-	-	1.13* (0.49)
Polity (1-21) × Initiator	-	-	-0.02 (0.10)	-	-
Polity (1-21) Squared × Initiator	-	-	0.00 (0.00)	-	-
Polity (1-21)	-	-	-	0.10* (0.05)	-
Polity (1 to 21) × Target	0.06* (0.03)	0.065* (0.28)	0.064# (0.036)	-	-
Initiation	0.96** (0.35)	0.81* (0.36)	1.16# (0.69)	-	-
Relative Capabilities	3.76*** (0.53)	3.75*** (0.53)	3.72*** (0.75)	4.74*** (1.30)	4.89*** (1.31)
Alliance Contribution	4.72*** (0.68)	4.74*** (0.68)	4.72*** (0.98)	4.11** (1.55)	4.00* (1.55)
Quality Ratio	0.05 (0.03)	0.05 (0.03)	0.06 (0.04)	0.21 (0.15)	0.20 (0.15)
Terrain	-11.31*** (3.02)	-10.92*** (2.95)	-10.80** (3.95)	-9.35*** (2.50)	-9.65*** (2.53)
Strategy × Terrain	3.68*** (0.99)	3.56*** (0.97)	3.52** (1.32)	2.08** (0.71)	2.19 (0.73)
Strategy 1	7.62** (2.93)	7.41** (2.92)	7.25# (3.99)	-	-
Strategy 2	3.76# (2.02)	3.55# (2.00)	3.47 (2.72)	-	-
Strategy 3	3.57* (1.45)	3.50* (1.45)	3.42# (1.96)	-	-
Strategy 4	3.21* (1.26)	3.22* (1.27)	3.15* (1.55)	-	-
Constant	-3.93* (1.57)	-3.87* (1.57)	-6.29** (2.30)	-0.05 (0.96)	1.30** (0.44)
N	197	197	197	86	86
Log-LL	-64.86	-64.59	-64.66	-21.29	-20.97
Wald Chi²	97.92***	95.82***	97.54***	68.66***	69.30***

# p < 0.10, * p < 0.05, ** p< 0.01, *** p< 0.001. In models 1 and 2, standard errors are robust but not clustered, to match Reiter and Stam's practice; in other model 3, standard errors are clustered by war number. In models 4 and 5, standard errors are not robust or clustered since only initiators are included.

Model 1: Deviance = 129.73; Powers = -0.5,-0.5
Model 2: Deviance = 129.19; Deviance for linear model = 129.77; Difference in Deviance = 0.58 (*p*-value = 0.90); Powers = 0,1
Model 3: Deviance = 129.32
Model 4: Deviance = 42.58
Model 5: Deviance = 41.94; Difference in Deviance from linear model = 0.64 (*p*-value = 0.89); Powers = -2,0

Figure 1. Post-Estimation Plot Showing Cluster of Observations at Zero Using Reiter and Stam's Fractional Polynomials from *Democracies at War*

Fractional Polynomial (-.5 -.5),
adjusted for covariates