COMPARING THE EFFECT OF LINEAR VERSUS CURVILINEAR SPECIFICATIONS OF DEMOCRACY ON WAR OUTCOMES[1]

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In table 2 of their letter to the editors of *International Security* regarding my article "How Smart and Tough Are Democracies? Reassessing Theories of Democratic Victory in War," Dan Reiter and Allan Stam present regression results that purportedly provide evidence of a curvilinear relationship between level of democracy and victory in war.[2] They take me to task in their note for failing to test for this curvilinear relationship, which they contend is the "central argument" of their book *Democracies at War*.[3] Because they do not accept my argument that draws belong in any analysis of war outcomes, Reiter and Stam omit these cases from their re-analysis and estimate a probit model. They also reject my decision to code the 1969 War of Attrition between Egypt and Israel as a draw, coding it instead as an Israeli win. However, they do not contest my choice to credit Israel with only one victory in the 1973 Yom Kippur War rather than two (Reiter and Stam had divided this conflict into two separate wars, thus crediting Israel with two wins). These differences aside, Reiter and Stam report that they use my codings for war outcome and initiator/target.

To model curvilinearity in the relationship between regime type and victory, Reiter and Stam use two fractional polynomial terms: $x^{-½} \times$ initiator and $x^{-½}(\ln(x)) \times$ initiator, where x is the Polity index of democracy (ranging in this case from 1 to 21 rather than -10 to +10).

[1] This note and the accompanying table are intended to supplement my contribution to Dan Reiter, Allan C. Stam, and Alexander B. Downes, "Correspondence: Another Skirmish in the Battle over Democracies and War," *International Security*, Vol. 34, No. 2 (Fall 2009).
[2] Alexander B. Downes, "How Smart and Tough Are Democracies? Reassessing Theories of Democratic Victory in War," *International Security*, Vol. 33, No. 4 (Spring 2009), pp. 9-51; and Reiter and Stam, "Correspondence."
[3] Dan Reiter and Allan C. Stam, *Democracies at War* (Princeton, N.J.: Princeton University Press, 2002).

Fractional polynomials (FPs for short) are sometimes used in place of the more familiar squared or cubed terms to test for non-linearity because the greater number of available transformations can result in better fit. Reiter and Stam do not explain why they chose this particular combination of terms; further analysis indicates that there are actually FPs that fit the data better, although the improvement is slight.[4] It is also unclear why Reiter and Stam's FP model contains an interaction term for democracy × target when the model logically cannot include the constituent term of this interaction for democracy (this would result in both linear and curvilinear specifications of democracy in the same regression) and also simply omits the other constituent term, war target.[5]

Model 1 in table 1 below shows my replication of the model that appears in table 2 of Reiter and Stam's letter. For reasons that I have been unable to discover, there are minor differences between their results and mine in the coefficients and standard errors for most variables, resulting in a few cases in reduction in statistical significance. No variable that was significant in their estimation becomes insignificant in mine, however, and the core results for the FPs are nearly identical with no reduction in significance. The log-likelihood statistic of my replication (-63.56) actually represents a tiny improvement over Reiter and Stam's version (-63.59).

Model 2 in table 1 shows the results when democracy is treated as linear. The two FPs are omitted, Polity (1 to 21), initiator, and target are included, as are the interaction terms Polity × initiator and Polity × target. If Reiter and Stam are correct that there is a curvilinear relationship between democracy and victory, the log-likelihood of this model should be worse (diverging more from zero) than the log-likelihood of the FP model. The reason—as I pointed out in my

[4] For this point, and more on FPs in general, see Alexander B. Downes, "Note on Curvilinearity," http://www.duke.edu/~downes/publications.htm.

[5] Recall that Reiter and Stam are using my coding scheme, which includes a third category of war participant (joiners) in addition to initiators and targets. Inserting target into the regression thus does not result in multicollinearity.

reply to Reiter and Stam—is that statistical significance of the FPs is only the starting point for assessing evidence of curvilinearity. The FPs must be statistically significant, but the explanatory power of the model also needs to be significantly better than the linear specification.[6] The log-likelihood of model 2 (-62.40), however, with democracy treated as linear, is *better than* that of model 1 (-63.56). The data therefore prohibit us from concluding that the relationship between democracy and victory is curvilinear rather than linear, even when draws are omitted from the analysis.

Models 3 and 4 in the table below show results from two ordered probit models when draws are included. Model 3 includes two FPs to test for curvilinearity in the relationship between democracy and victory, whereas model 4 treats democracy as linear.[7] The first thing to notice is that the FPs in model 3 are no longer statistically significant. Moreover, once again the log-likelihood of the linear specification (-168.46) is greater than that of the curvilinear specification (-169.15), providing no evidence that modeling democracy in a non-linear fashion explains war outcomes better than treating democracy as linear.

Critics might point out that because the two linear models each have one more variable than the non-linear ones, it is no surprise that the former out-perform the latter. To check for this possibility, I dropped both target and Polity × target from models 2 and 4, thereby giving the curvilinear models an advantage.[8] Even with this leg up, the linear models still explain more of

[6] Patrick Royston and Willi Sauerbrei, *Multivariable Model-Building: A Pragmatic Approach to Regression Analysis Based on Fractional Polynomials for Modelling Continuous Covariates* (Chichester, U.K.: John Wiley, 2008), pp. 82-83. I discuss this issue in more detail in Downes, "Note on Curvilinearity."

[7] Model 4 is identical to model 3 in my original article; Downes, "How Smart and Tough Are Democracies?" pp. 20-21.

[8] The better comparison would be to drop any variables dealing with targets from both models, allowing for a direct comparison of linearity vs. curvilinearity with regard to war initiators. Doing so would drop Polity × target from models 1 and 3, and target and Polity × target from models 2 and 4. Performing the analysis this way leads to slightly worse results for the curvilinear specifications than those reported above, with log-likelihoods of -62.99 (linear) vs. -63.67 (curvilinear) without draws, and -168.74 vs. -169.23 with draws.

the variation in war outcomes than the curvilinear ones: the respective log-likelihoods are -62.99 vs. -63.56 without draws, and -168.74 vs. -169.15 with draws.

I conclude from this analysis that there is no evidence that treating democracy as curvilinear is a statistically significant improvement over modeling it as linear. Thus I was correct to analyze the linear effect of democracy on the probability of victory in my article.

Table 1. Linear versus Curvilinear Specifications of Democracy, Initiation, and War Outcomes

	1 Replication of Reiter and Stam, "Correspondence," Table 2; DV = win/lose (probit)	2 Linear democracy terms substituted for fractional polynomials; DV = win/lose (probit)	3 Same as Model 1, draws added (ordered probit)	4 Same as Model 2, draws added (ordered probit)
Initiation	6.11** (1.76)	1.00 (0.73)	2.10 (1.43)	0.37 (0.56)
Target	-	0.56 (0.79)	-	-0.16 (0.60)
Fractional Term 1	-4.77** (1.65)	-	-1.33 (1.23)	-
Fractional Term 2	-4.72** (1.68)	-	-1.52 (1.41)	-
Polity (1 to 21)	-	0.10* (0.05)	-	0.03 (0.045)
Polity (1 to 21) × Initiator	-	-0.02 (0.05)	-	-0.009 (0.047)
Polity (1 to 21) × Target	0.01 (0.02)	-0.08 (0.05)	0.01 (0.02)	-0.008 (0.048)
Relative Capabilities	3.88*** (0.75)	3.88*** (0.78)	2.39*** (0.49)	2.35*** (0.49)
Alliance Contribution	4.76*** (1.11)	4.71*** (1.19)	3.09*** (0.75)	3.00*** (0.74)
Quality Ratio	0.05 (0.04)	0.06 (0.04)	0.05# (0.02)	0.04# (0.02)
Terrain	-13.65*** (3.76)	-13.31*** (3.62)	-1.97# (1.17)	-1.89# (1.13)
Strategy × Terrain	4.36*** (1.23)	4.30*** (1.16)	0.53 (0.38)	0.50 (0.36)
Strategy 1	9.37** (3.57)	9.11** (3.45)	-0.38 (1.35)	-0.47 (1.30)
Strategy 2	4.36# (2.61)	4.14# (2.50)	-2.53** (0.92)	-2.55** (0.88)
Strategy 3	4.42* (1.76)	4.20* (1.71)	-0.13 (0.68)	-0.21 (0.65)
Strategy 4	3.58* (1.39)	3.59* (1.36)	1.16 (0.76)	1.11 (0.73)
Constant	-6.73** (2.01)	-6.58** (1.91)	-	-
N	196	196	233	233
Log-LL	-63.56	-62.40	-169.15	-168.46
Wald Chi^2	95.10***	93.23***	98.09***	95.89***

Robust standard errors clustered on each war in parentheses; # $p < 0.10$, * $p < 0.05$, ** $p< 0.01$, *** $p< 0.001$